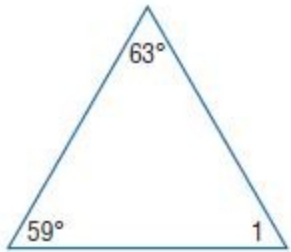


4-2 Angles of Triangles

Find the measures of each numbered angle.

1.

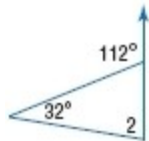


ANSWER:

58

Find each measure.

3. $m\angle 2$

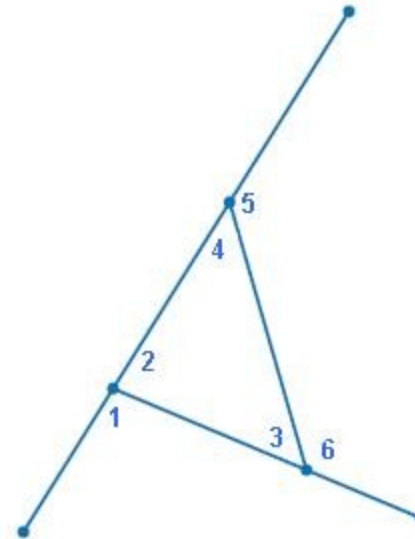


ANSWER:

80

DECK CHAIRS The brace of this deck chair forms a triangle with the rest of the chair's frame as shown. If $m\angle 1 = 102$ and $m\angle 3 = 53$, find each measure.

Refer to the figure on page 250.



5. $m\angle 4$

ANSWER:

49

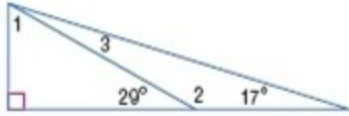
7. $m\angle 2$

ANSWER:

78

4-2 Angles of Triangles

CCSS REGULARITY Find each measure.



9. $m\angle 1$

ANSWER:

61

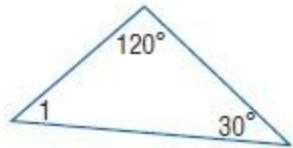
11. $m\angle 2$

ANSWER:

151

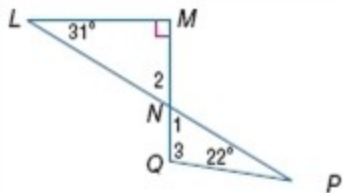
Find the measure of each numbered angle.

13. Refer to the figure on page 250.



ANSWER:

30



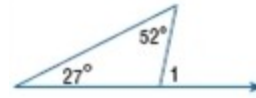
15.

ANSWER:

$$m\angle 1 = 59, m\angle 2 = 59, m\angle 3 = 99$$

Find each measure.

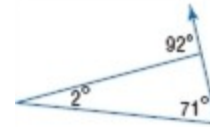
17. $m\angle 1$



ANSWER:

79

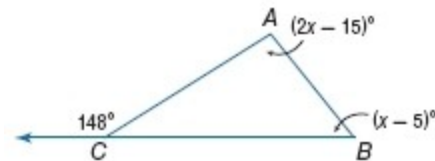
19. $m\angle 2$



ANSWER:

21

21. $m\angle ABC$

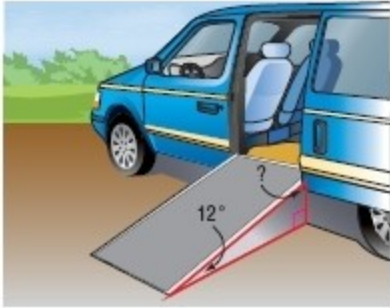


ANSWER:

51

4-2 Angles of Triangles

23. **WHEELCHAIR RAMP** Suppose the wheelchair ramp shown makes a 12° angle with the ground. What is the measure of the angle the ramp makes with the van door?



ANSWER:

78

CCSS REGULARITY Find each measure.

25. $m\angle 2$

ANSWER:

39

27. $m\angle 4$

ANSWER:

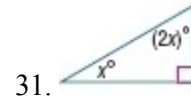
55

29. $m\angle 6$

ANSWER:

35

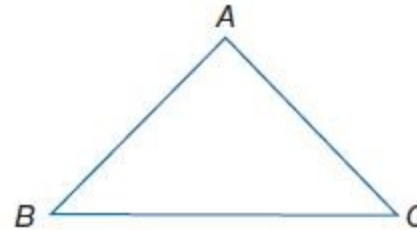
ALGEBRA Find the value of x . Then find the measure of each angle.



ANSWER:

$x = 30$; 30, 60, 90

33. **GARDENING** A landscaper is forming an isosceles triangle in a flowerbed using chrysanthemums. She wants $m\angle A$ to be three times the measure of $\angle B$ and $\angle C$. What should the measure of each angle be?



ANSWER:

$m\angle A = 108$, $m\angle B = m\angle C = 36$

4-2 Angles of Triangles

PROOF Write the specified type of proof.

35. paragraph proof of Corollary 4.2

ANSWER:

Given: $\triangle MNO$

$\angle M$ is a right angle.

Prove: There can be at most one right angle in a triangle.

Proof: In $\triangle MNO$ M is a right angle. $m\angle M + m\angle N + m\angle O = 180$.

$m\angle M = 90$, so $m\angle N + m\angle O = 90$.

If N were a right angle, then $m\angle O = 0$. But that is impossible, so there cannot be two right angles in a triangle.

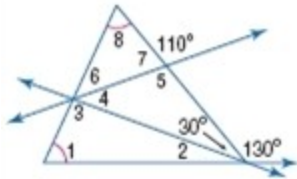
Given: $\triangle PQR$

$\angle P$ is obtuse.

Prove: There can be at most one obtuse angle in a triangle.

Proof: In $\triangle PQR$, $\angle P$ is obtuse. So $m\angle P > 90$. $m\angle P + m\angle Q + m\angle R = 180$. It must be that $m\angle Q + m\angle R < 90$. So, $\angle Q$ and $\angle R$ must be acute.

CCSS REGULARITY Find the measure of each numbered angle.



37.

ANSWER:

$m\angle 1 = 65$, $m\angle 2 = 20$, $m\angle 3 = 95$, $m\angle 4 = 40$, $m\angle 5 = 110$, $m\angle 6 = 45$,

$m\angle 7 = 70$, $m\angle 8 = 65$

39. **ALGEBRA** The measure of the larger acute angle in a right triangle is two degrees less than three times the measure of the smaller acute angle. Find the measure of each angle.

ANSWER:

67° , 23°

41. **ALGEBRA** In $\triangle XYZ$, $m\angle X = 157$, $m\angle Y = y$, and $m\angle Z = z$. Write an inequality to describe the possible measures of $\angle Z$. Explain your reasoning.

ANSWER:

$z < 23$; Sample answer: Since the sum of the measures of the angles of a triangle is 180 and $m\angle X = 157$, $157 + m\angle Y + m\angle Z = 180$, so $m\angle Y + m\angle Z = 23$. If $m\angle Y$ was 0, then $m\angle Z$ would equal 23. But since an angle must have a measure greater than 0, $m\angle Z$ must be less than 23, so $z < 23$.

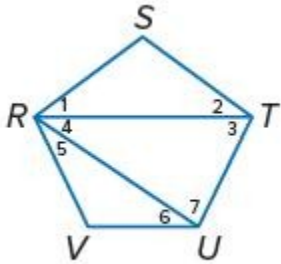
4-2 Angles of Triangles

PROOF Write the specified type of proof.

43. two-column proof

Given: $RSTUV$ is a pentagon.

Prove: $m\angle S + m\angle STU + m\angle TUV + m\angle V + m\angle VRS = 540$



ANSWER:

Proof: Statements (Reasons)

1. $RSTUV$ is a pentagon. (Given)

2. $m\angle S + m\angle 1 + m\angle 2 = 180$;

$m\angle 3 + m\angle 4 + m\angle 7 = 180$;

$m\angle 6 + m\angle V + m\angle 5 = 180$ (\angle Sum Thm.)

3.

$m\angle S + m\angle 1 + m\angle 2 +$

$m\angle 3 + m\angle 4 + m\angle 7 +$

$m\angle 6 + m\angle V + m\angle 5 = 540$ (Add. Prop.)

4. $m\angle VRS = m\angle 1 + m\angle 4 + m\angle 5$;

$m\angle TUV = m\angle 7 + m\angle 6$;

$m\angle STU = m\angle 2 + m\angle 3$ (\angle Addition)

5.

$m\angle S + m\angle STU + m\angle TUV +$

$m\angle V + m\angle VRS = 540$ (Subst.)

45. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the sum of the measures of the exterior angles of a triangle.

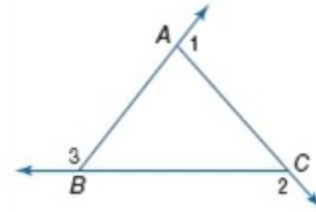
a. GEOMETRIC Draw five different triangles, extending the sides and labeling the angles as shown. Be sure to include at least one obtuse, one right, and one acute triangle.

b. TABULAR Measure the exterior angles of each triangle. Record the measures for each triangle and the sum of these measures in a table.

c. VERBAL Make a conjecture about the sum of the exterior angles of a triangle. State your conjecture using words.

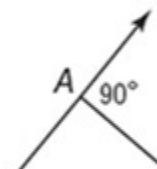
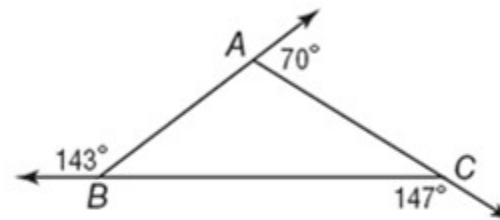
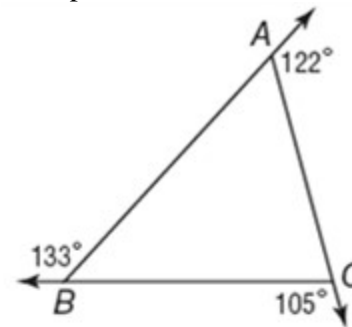
d. ALGEBRAIC State the conjecture you wrote in part c algebraically.

e. ANALYTICAL Write a paragraph proof of your conjecture.

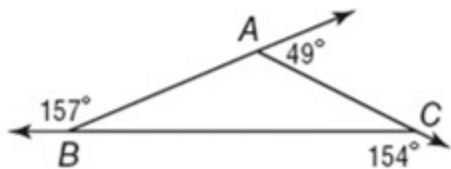
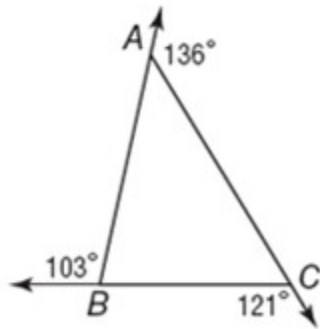


ANSWER:

a. Sample answer:



4-2 Angles of Triangles



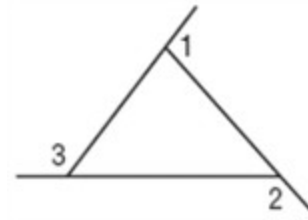
b. Sample answer

$\angle 1$	$\angle 2$	$\angle 3$	Sum
122	105	133	360
70	147	143	360
90	140	130	360
136	121	103	360
49	154	157	360

c. Sample answer:

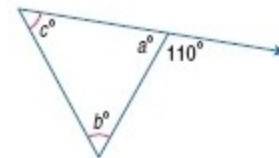
The sum of the measures of the exterior angles of a triangle is 360.

d. $m\angle 1 + m\angle 2 + m\angle 3 = 360$



- e. The Exterior Angle Theorem tells us that $m\angle 3 = m\angle BAC + m\angle BCA$,
 $m\angle 2 = m\angle BAC + m\angle CBA$,
 $m\angle 1 = m\angle CBA + m\angle BCA$.
 Through substitution,
 $m\angle 1 + m\angle 2 + m\angle 3 = m\angle CBA + m\angle BCA + m\angle BAC + m\angle CBA + m\angle BAC + m\angle BCA$. Which can be simplified to $m\angle 1 + m\angle 2 + m\angle 3 = 2m\angle BAC + 2m\angle BCA + 2m\angle CBA$.
 The Distributive Property can be applied and gives $m\angle 1 + m\angle 2 + m\angle 3 = 2(m\angle BAC + m\angle BCA + m\angle CBA)$. The Triangle Angle-Sum Theorem tells us that $m\angle BAC + m\angle BCA + m\angle CBA = 180$. Through substitution we have $m\angle 1 + m\angle 2 + m\angle 3 = 2(180) = 360$.

47. **WRITING IN MATH** Explain how you would find the missing measures in the figure shown.

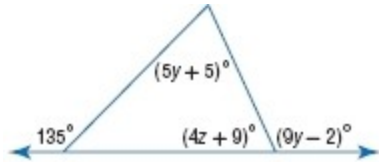


ANSWER:

The measure of $\angle a$ is the supplement of the exterior angle with measure 110, so $m\angle a = 180 - 110$ or 70. Because the angles with measures b and c are congruent, $b = c$. Using the Exterior Angle Theorem, $b + c = 110$. By substitution, $b + b = 110$, so $2b = 110$ and $b = 55$. Because $b = c$, $c = 55$.

4-2 Angles of Triangles

49. **CHALLENGE** Find the values of y and z in the figure.



ANSWER:

$$y = 13, z = 14$$

51. **WRITING IN MATH** Explain why a triangle cannot have an obtuse, acute, and a right exterior angle.

ANSWER:

Sample answer: Since an exterior angle is acute, the adjacent angle must be obtuse. Since another exterior angle is right, the adjacent angle must be right. A triangle cannot contain both a right and an obtuse angle because it would be more than 180 degrees. Therefore, a triangle cannot have an obtuse, acute, and a right exterior angle.

53. **SHORT RESPONSE** Two angles of a triangle have measures of 35° and 80° . Describe the possible values of the exterior angle measures of the triangle.

ANSWER:

$$100^\circ, 115^\circ, 145^\circ.$$

55. **SAT/ACT** Joey has 4 more video games than Solana and half as many as Melissa. If together they have 24 video games, how many does Melissa have?

- A 7
- B 9
- C 12
- D 13
- E 14

ANSWER:

E

Classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*.



ANSWER:

obtuse

COORDINATE GEOMETRY Find the distance from P to ℓ .

59. Line ℓ contains points $(0, -2)$ and $(1, 3)$. Point P has coordinates $(-4, 4)$.

ANSWER:

$$\sqrt{26} \text{ units.}$$

Write a conjecture that describes the pattern in each sequence. Then use your conjecture to find the next item in the sequence.



ANSWER:

Each set of figures has one more triangle than the previous set and the direction of the triangles alternate between pointing up and pointing to the right;



State the property that justifies each statement.

63. If $\frac{x}{2} = 7$, then $x = 14$.

ANSWER:

Multiplication Property

4-2 Angles of Triangles

65. If $XY - AB = WZ - AB$, then $XY = WZ$.

ANSWER:

Addition Property

67. If $m\angle 1 + m\angle 2 = 90$ and $m\angle 2 = m\angle 3$, then $m\angle 1 + m\angle 3 = 90$.

ANSWER:

Substitution Property