

Question	Answer	Solution
1.	Yes	Yes, the set is closed: $0 \cdot 0 = 0$ ; $0 \cdot 1 = 0$ , and $1 \cdot 1 = 1$ .
2.	No	No, the set is not closed: $1 + 1 = 2$ , and 2 is not a number in the set.
3.	Yes	Yes, the set is closed: for example, $2 + 4 = 6$ (an even integer); in general, if $m$ is an integer, $2m$ is an even integer (because even integers are multiples of 2); similarly, if $n$ is an integer, $2n$ is an even integer. The sum $2m + 2n = 2(m + n)$ is an even integer, since $(m + n)$ is an integer.
4.	Let $a$ and $b$ be rational numbers. Then $a = \frac{p}{q}$ and $b = \frac{r}{s}$ , where $p$ , $q$ , $r$ , and $s$ are integers and $q$ and $s$ are not 0. $a \times b = \frac{p}{q} \times \frac{r}{s} = \frac{pr}{qs}$ Because $pr$ and $qs$ are integers, $\frac{pr}{qs}$ is a rational number. So the set of rational numbers is closed under multiplication.	Let $a$ and $b$ be rational numbers. Then $a = \frac{p}{q}$ and $b = \frac{r}{s}$ , where $p$ , $q$ , $r$ , and $s$ are integers and $q$ and $s$ are not 0. $a \times b = \frac{p}{q} \times \frac{r}{s} = \frac{pr}{qs}$ Because $pr$ and $qs$ are integers, $\frac{pr}{qs}$ is a rational number. So the set of rational numbers is closed under multiplication.
5.	Let $a$ be an irrational number and let $b$ be a nonzero rational number. Then $b = \frac{r}{s}$ , where $r$ and $s$ are integers and $r$ and $s$ are not 0. The product $a \times b$ must be either rational or irrational. Assume that the product $a \times b$ is rational. Then $a \times b = \frac{p}{q}$ , where $p$ and $q$ are integers and $q$ is not 0. $a \times b = \frac{p}{q}$ $a \times b \cdot \frac{1}{b} = \frac{p}{q} \cdot \frac{1}{b} \quad (b \neq 0)$ $a = \frac{p}{q} \div b \quad (b \neq 0)$ $a = \frac{p}{q} \div \frac{r}{s} \quad (s \neq 0)$ $a = \frac{p}{q} \times \frac{s}{r} \quad (r \neq 0)$ $a = \frac{ps}{qr}$ The final equation shows that $a$ is a rational number. But $a$ is given as an irrational number. Therefore, the assumption the product $a \times b$ is rational is incorrect. So the product of an irrational number and a nonzero rational number is irrational.	Let $a$ be an irrational number and let $b$ be a nonzero rational number. Then $b = \frac{r}{s}$ , where $r$ and $s$ are integers and $r$ and $s$ are not 0. The product $a \times b$ must be either rational or irrational. Assume that the product $a \times b$ is rational. Then $a \times b = \frac{p}{q}$ , where $p$ and $q$ are integers and $q$ is not 0. $a \times b = \frac{p}{q}$ $a \times b \cdot \frac{1}{b} = \frac{p}{q} \cdot \frac{1}{b} \quad (b \neq 0)$ $a = \frac{p}{q} \div b \quad (b \neq 0)$ $a = \frac{p}{q} \div \frac{r}{s} \quad (s \neq 0)$ $a = \frac{p}{q} \times \frac{s}{r} \quad (r \neq 0)$ $a = \frac{ps}{qr}$ The final equation shows that $a$ is a rational number. But $a$ is given as an irrational number. Therefore, the assumption the product $a \times b$ is rational is incorrect. So the product of an irrational number and a nonzero rational number is irrational.
6.	Find an example that shows that when the operation is applied to numbers in the set, the result is a number that is not in the set. In other words, find at least one	Find an example that shows that when the operation is applied to numbers in the set, the result is a number that is not in the set. In other words, find at least one

Question	Answer	Solution
7.	Sample answer: -1, and -5	Sample answer: -1, and -5
8.	Sample answer: $-\frac{1}{2}$ and $-\frac{3}{4}$	Sample answer: $-\frac{1}{2}$ and $-\frac{3}{4}$
9.	none	none
10.	Sample answer: $-\sqrt[3]{5}$ and $-\pi$	Sample answer: $-\sqrt[3]{5}$ and $-\pi$
11.	15: whole, integer, rational	15: whole, integer, rational
12.	irrational	$\frac{\pi}{2}$ : irrational
13.	integer, rational	$-\frac{\sqrt[3]{8}}{2} = -1$ : integer, rational
14.	whole, integer, rational	$\frac{\sqrt{36}}{\sqrt{9}} = \frac{6}{3} = 2$ : whole, integer, rational
15.	whole, integer, rational	$\frac{0}{\sqrt{7}} = 0$ : whole, integer, rational
16.	The set is not closed; $-1 - (-3) = 2$ , and 2 is not a negative integer.	The set is not closed; $-1 - (-3) = 2$ , and 2 is not a negative integer.
17.	The set is closed.	The set is closed.
18.	The set is closed.	The set is closed.
19.	The set is not closed: $2 \times 2 = 4$ , and 4 is not a number in the set.	The set is not closed: $2 \times 2 = 4$ , and 4 is not a number in the set.
20.	The set is not closed: $-1 \times (-3) = 3$ , and 3 is not a negative integer.	The set is not closed: $-1 \times (-3) = 3$ , and 3 is not a negative integer.
21.	The set is not closed: $-\frac{1}{2} \times \left(-\frac{1}{2}\right) = \frac{1}{4}$ , and $\frac{1}{4}$ is not a negative integer.	The set is not closed: $-\frac{1}{2} \times \left(-\frac{1}{2}\right) = \frac{1}{4}$ , and $\frac{1}{4}$ is not a negative integer.
22.	The set is closed.	The set is closed.
23.	The set is not closed: $\sqrt{5} \times \sqrt{5} = 5$ , and 5 is not a positive irrational number.	The set is not closed: $\sqrt{5} \times \sqrt{5} = 5$ , and 5 is not a positive irrational number.
24.	The set is not closed: $10 \times 10 = 100$ , and 100 is not a number in the set.	The set is not closed: $10 \times 10 = 100$ , and 100 is not a number in the set.
25.	The set is closed.	The set is closed.